3. Bose Einstein's Distribution Law

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The Bose-Einstein Distribution

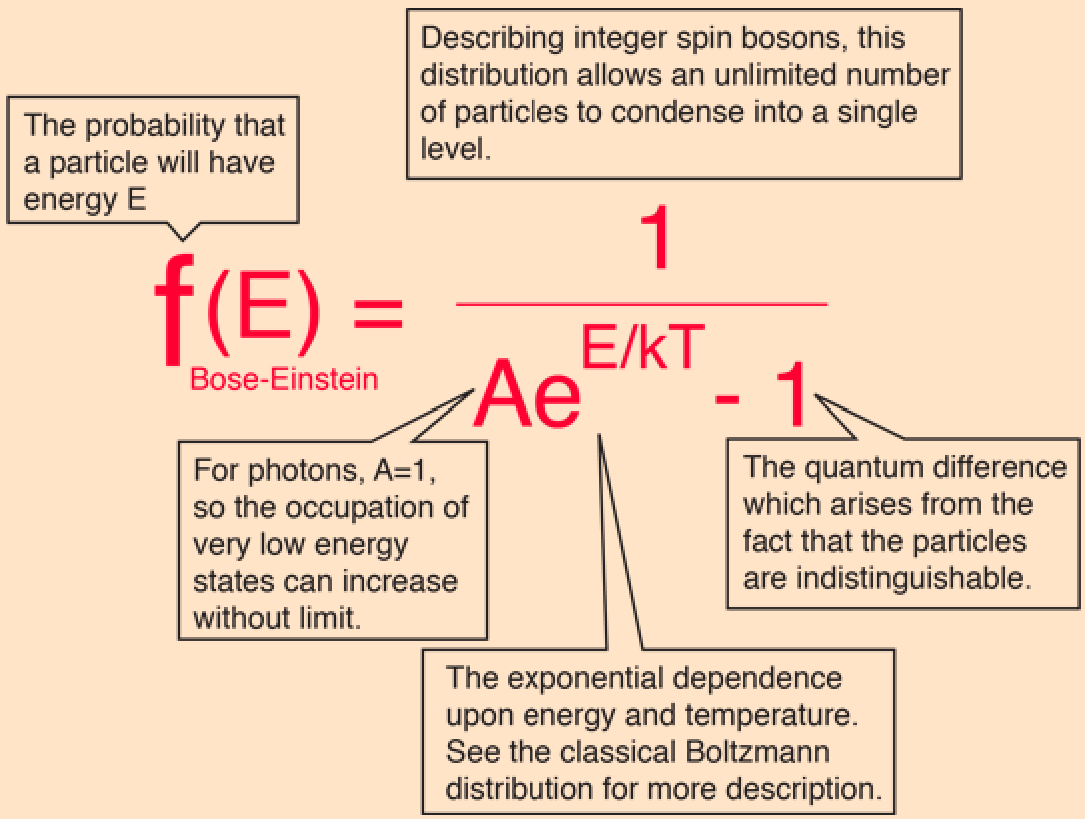
The Bose-Einstein distribution describes the statistical behavior of integer spin particles (bosons). At low temperatures, bosons can behave very differently than fermions because an unlimited number of them can collect into the same energy state, a phenomenon called "condensation".

$$f(E) = \frac{1}{Ae^{E/kT} - 1}$$

Explain the symbols

Bose-Einstein statistics, one of two possible ways in which a collection of indistinguishable particles may occupy a set of available discrete energy states. The aggregation of particles in the same state, which is characteristic of particles obeying Bose-Einstein statistics, accounts for the cohesive streaming of laser light and the frictionless creeping of superfluid helium. The theory of this behaviour was developed (1924–25) by Albert Einstein and the Indian physicist Satyendra Nath Bose, who recognized that a collection of identical and indistinguishable particles can be distributed in this way.

Bose-Einstein Details



In contrast to Fermi-Dirac statistics, the Bose-Einstein statistics apply only to those particles not limited to single occupancy of the same state—that is, particles that do not obey the restriction known as the Pauli exclusion principle. Such particles have integer values of spin and are named bosons, after the statistics that correctly describe their behaviour.

States of Matter: Bose-Einstein Condensate

An illustration of a "super-photon" created when physicists turned photons of light into a state of matter called a Bose-Einstein condensate. (Image credit: Jan Klaers, University of Bonn)

Of the five states matter can be in, the Bose-Einstein condensate is perhaps the most mysterious. Gases, liquids, solids and plasmas were all well studied for decades, if not centuries; Bose-Einstein condensates weren't created in the laboratory until the 1990s.

A Bose-Einstein condensate is a group of atoms cooled to within a hair of absolute zero. When they reach that temperature the atoms are hardly moving relative to each other; they have almost no free energy to do so. At that point, the atoms begin to clump together, and enter the same energy states. They become identical, from a physical point of view, and the whole group starts behaving as though it were a single atom.

To make a Bose-Einstein condensate, you start with a cloud of diffuse gas. Many experiments start with atoms of rubidium. Then you cool it with lasers, using the beams to take energy away from the atoms. After that, to cool them further, scientists use evaporative cooling. "With a [Bose-Einstein condensate], you start from a disordered state, where kinetic energy is greater than potential energy," said Xuedong Hu, a professor of physics at the University at Buffalo. "You cool it down, but it doesn't form a lattice like a solid."

Instead, the atoms fall into the same quantum states, and can't be distinguished from one another. At that point the atoms start obeying what are called Bose-Einstein statistics, which are usually applied to particles you can't tell apart, such as photons.

Theory & discovery

Bose-Einstein condensates were first predicted theoretically by Satyendra Nath Bose (1894-1974), an Indian physicist who also discovered the subatomic particle named for him, the boson. Bose was working on statistical problems in quantum mechanics, and sent his ideas to Albert Einstein. Einstein thought them important enough to get them published. As importantly, Einstein saw that Bose's mathematics — later known as Bose-Einstein statistics — could be applied to atoms as well as light.

What the two found was that ordinarily, atoms have to have certain energies — in fact one of the fundamentals of quantum mechanics is that the energy of an atom or other subatomic particle can't be arbitrary. This is why electrons, for example, have discrete "orbitals" that they have to occupy, and why they give off photons of specific wavelengths when they drop from one orbital, or energy level, to another. But cool the atoms to within billionths of a degree of absolute zero and some atoms begin to fall into the same energy level, becoming indistinguishable.

That's why the atoms in a Bose-Einstein condensate behave like "super atoms." When one tries to measure where they are, instead of seeing discrete atoms one sees more of a fuzzy ball.

Other states of matter all follow the Pauli Exclusion Principle, named for physicist Wolfgang Pauli. Pauli (1900-1958) was an Austrian-born Swiss and American theoretical physicist and one of the pioneers of quantum physics.It says that fermions — the kinds of particles that make up matter — can't be in identical quantum states. This is why when two electrons are in the same orbital, their spins have to be opposite so they add up to zero. That in turn is one reason why chemistry works the way it does and one reason atoms can't occupy the same space at the same time. Bose-Einstein condensates break that rule.

Though the theory said such states of matter should exist, it wasn't until 1995 that Eric A. Cornell and Carl E. Wieman, both of the Joint Institute for Lab Astrophysics (JILA) in Boulder, Colorado, and Wolfgang Ketterle, of the Massachusetts Institute of Technology, managed to make one, for which they got the 2001 Nobel Prize in Physics.

Let 'n' be the total no. of bosons in the system. Let n_1, n_2… n_i be the no. of particles with energy E_1, E_2… E_i respectively. Let gi be the no. of quantum states for the energy level E_i. The quantity g_i is called the degeneracy or the multiplicity of the energy level E_i.

The no. of bosons which can occupy the energy level E_i having degeneracy g_i is given as - $n_i = \frac{g_i}{e^{\alpha+\beta\varepsilon_i} - 1}$ or $\frac{n_i}{g_i} = \frac{1}{e^{\alpha+\beta\varepsilon_i} - 1}$

But $\frac{n_i}{g_i} = f(E_i)$ so $\boxed{f(E_i) = \frac{1}{e^{\alpha+\beta\varepsilon_i} - 1}}$ -------------- (1)

Equation (1) is known as B – E distribution function. The quantities 'α' and 'β' are called Lagrange's multiplier where β = 1/kT and 'α' is determined according to physical state of the system.

Application of B – E Statistics to Photon Gas

OR

Planck's Radiation Law (Planck's Oscillator): Planck's radiation law tells us about the distribution of energy in black body radiations. According to Planck's law the energy density $E_\upsilon d\upsilon$lying in the frequency range υ and υ + dυ is given as –

$E_\upsilon d\upsilon = \frac{8\pi h \upsilon^3}{c^3}\frac{d\upsilon}{e^{h\upsilon/kT}-1}$ or in terms of wavelength $E_\lambda d\lambda = \frac{8\pi h c}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$ which is the mathematical statement of Planck's law.

Proof: Let's consider a black body at a temperature 'T'. The radiation emitted by it consists of photons of energy hυ and momentum hυ/c. According to B – E statistics, no. of photons having energy E_i and degeneracy g_i is given by - $n_i = \frac{g_i}{e^{\alpha+\beta\varepsilon_i}-1}$. Therefore, no. of photons lying between frequency υ and υ + dυ is given by –

$$n_\upsilon d\upsilon = \frac{g_\upsilon d\upsilon}{e^{\alpha+(h\upsilon/kT)}-1} \quad \text{-------------- (1)}$$

According to Planck, exchange of energy between photon takes in such a way that –

(i) Total energy of all the photons is constant i.e. E = $\sum_i n_i E_i = \text{Constant}$

(ii) Total no. of photons do not remains constant because it may happen that a photon of energy 2hυ is absorbed and two photons each of hυ may be emitted. This makes Lagrange's multiplier α = 0. So equation (1) becomes -

$$n_\upsilon d\upsilon = \frac{g_\upsilon d\upsilon}{e^{h\upsilon/kT}-1} \quad \text{-------------- (2)}$$

where g_υ dυ is the no. of quantum states lying between frequency range υ and υ+dυ.

In the momentum space, no. of quantum states per unit volume is = v/h^3. if Δv_p be the volume of momentum space lying between p and p + dp, then Δv_p will be the volume of spherical shells of radius p and thickness dp. So,

$\Delta v_p = 4\pi p^2 dp$

Therefore, total no. of quantum states lying between p and p + dp are given as –

$g_p\, dp = \frac{V}{h^3} 4\pi p^2 dp$

for a photon – $p = h\upsilon/c$ so - $dp = h/c\, d\upsilon$

Therefore, total no. of quantum states lying between υ and $\upsilon + d\upsilon$ are given as –

$$g_\upsilon d\upsilon = \frac{4\pi V}{h^3}\frac{h^2\upsilon^2}{c^2}\frac{h}{c}d\upsilon \quad \Rightarrow \quad g_\upsilon d\upsilon = \frac{4\pi V \upsilon^2}{c^3}d\upsilon \qquad \text{-------------- (3)}$$

Since there may be two types of photon – having left hand polarization as well as right hand polarization, so R.H.S. of equation (3) should be multiplied by 2. So –

$$g_\upsilon d\upsilon = \frac{8\pi V \upsilon^2}{c^3}d\upsilon \qquad \text{-------------- (4)}$$

Putting this value in equation (2) –

$$n_\upsilon d\upsilon = \frac{8\pi V \upsilon^2}{c^3}\frac{d\upsilon}{e^{h\upsilon/kT}-1} \qquad \text{-------------- (5)}$$

Since energy of one photon = $h\upsilon$. Hence total energy of photons lying between frequency range υ and $\upsilon + d\upsilon$ is = $h\upsilon\, n_\upsilon d\upsilon$ = $\frac{8\pi V \upsilon^2}{c^3}\frac{d\upsilon}{e^{h\upsilon/kT}-1}\times h\upsilon$

Now energy density $E_\upsilon\, d\upsilon = \frac{\text{Total Energy}}{\text{Volume}} = \frac{\frac{8\pi V \upsilon^2}{c^3}\frac{d\upsilon}{e^{h\upsilon/kT}-1}\times h\upsilon}{V}$

Or $$\boxed{E_\upsilon\, d\upsilon = \frac{8\pi h \upsilon^3}{c^3}\frac{d\upsilon}{e^{h\upsilon/kT}-1}} \qquad \text{-------------- (6)}$$

Equation (6) is called Planck's radiation law.