Fermi Level and Fermi Energy

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Fermi Level

"Fermi level" is the term used to describe the top of the collection of electron energy levels at absolute zero temperature. This concept comes from Fermi-Dirac statistics. Electrons are fermions and by the Pauli exclusion principle cannot exist in identical energy states. So at absolute zero they pack into the lowest available energy states and build up a "Fermi sea" of electron energy states. The Fermi level is the surface of that sea at absolute zero where no electrons will have enough energy to rise above the surface. The concept of the Fermi energy is a crucially important concept for the understanding of the electrical and thermal properties of solids. Both ordinary electrical and thermal processes involve energies of a small fraction of an electron volt. But the Fermi energies of metals are on the order of electron volts. This implies that the vast majority of the electrons cannot receive energy from those processes because there are no available energy states for them to go to within a fraction of an electron volt of their present energy. Limited to a tiny depth of energy, these interactions are limited to "ripples on the Fermi sea".

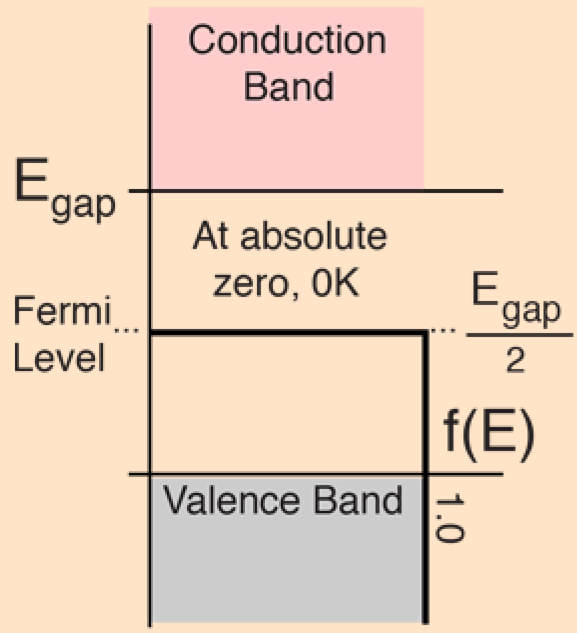
At higher temperatures a certain fraction, characterized by the Fermi function, will exist above the Fermi level. The Fermi level plays an important role in the band theory of solids. In doped semiconductors, p-type and n-type, the Fermi level is shifted by the impurities, illustrated by their band gaps. The Fermi level is referred to as the electron chemical potential in other contexts.

In metals, the Fermi energy gives us information about the velocities of the electrons which participate in ordinary electrical conduction. The amount of energy which can be given to an electron in such conduction processes is on the order of micro-electron volts, so only those electrons very close to the Fermi energy can participate. The Fermi velocity of these conduction electrons can be calculated from the Fermi energy.

$$v_F = \sqrt{\frac{2E_F}{m}}$$

Table

This speed is a part of the microscopic Ohm's Law for electrical conduction. For a metal, the density of conduction electrons can be implied from the Fermi energy.

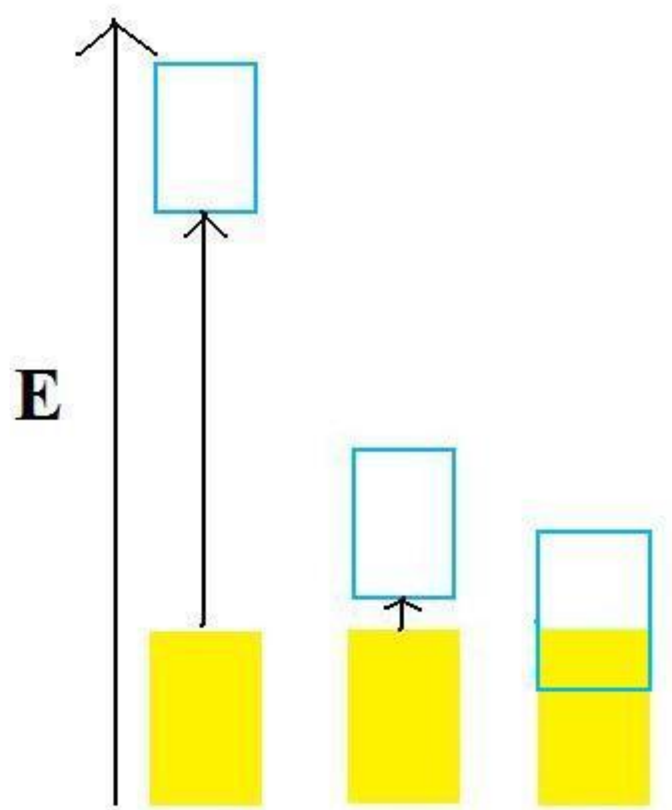


Context of Fermi level for a semiconductor

The Fermi energy also plays an important role in understanding the mystery of why electrons do not contribute significantly to the specific heat of solids at ordinary temperatures, while they are dominant contributors to thermal conductivity and electrical conductivity. Since only a tiny fraction of the electrons in a metal are within the thermal energy kT of the Fermi energy, they are "frozen out" of the heat capacity by the Pauli principle. At very low temperatures, the electron specific heat becomes significant.

Energy Band Diagram

The diagram below is an example of an energy band diagram. The upper box represents the conduction band, the lower box represents the valence band, and the yellow is the occupancy level of electrons. The conductivity of a material can be determined by the energy diagram. In order to conduct electricity, an electron must make a transition into another state since electrons cannot occupy the same quantum states. When the electron cannot make that transition, it will be not be able to conduct electricity. When the band is completely filled, the closest state an electron to transfer to will be the states in the conduction band, and that would require the electron to jump over the band gap, thus making it more difficult to conduct electricity. With this rule, it is easy to tell the difference between insulator, semi-conductor, and conductors. The energy band on the left side is an insulator because if an electron wants to go into a higher energy state, it will need to jump through that huge energy gap. Since it requires a large amount of energy to move the electron, the material will have a difficult time conducting electricity. The energy band in the middle is a semi-conductor because although the electrons have to jump across the energy gap, the energy gap is small. If an electron wants to make a transition, it will require very little energy since they are all int he same energy band. Thus, for the material in the middle, although it will have difficulty conducting electricity, it is not impossible. The energy band in the right is a conductor. Although the valence band is filled just like the insulator and semi-conductor, the conduction band overlaps the valence band. If an electron wants to make a transition, it will require very little amount of energy. Which means the material on the right can conduct electricity very easily.

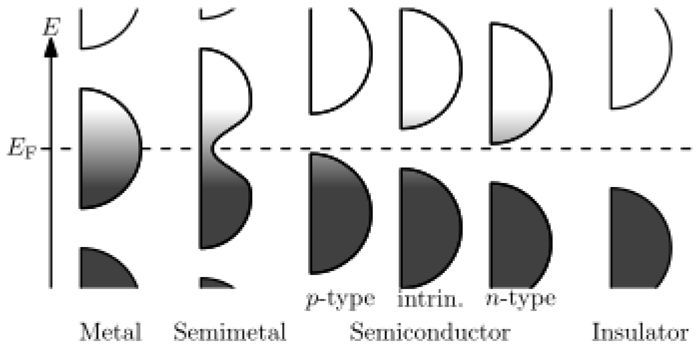


Fermi energy

Fermi energy is often defined as the highest occupied energy level of a material at absolute zero temperature. In other words, all electrons in a body occupy energy states at or below that body's Fermi energy at 0K.

The fermi energy is the difference in energy, mostly kinetic. In metals this means that it gives us the velecity of the electrons during conduction. So during the conduction process, only electrons that have an energy that is close to that of the fermi energy can be involved in the process.

This concept of Fermi energy is useful for describing and comparing the behaviour of different semiconductors. For example: an n-type semiconductor will have a Fermi energy close to the conduction band, whereas a p-type semiconductor will have a Fermi energy close to the valence band.



Fermi energies of different material types.

As a material's temperature rises above absolute zero, the probability of electrons existing in an energy state greater than the Fermi energy increases, and there is no longer any constant highest occupied level, so while the material's Fermi energy may be useful as a reference, it is not very useful at real temperatures.

Instead, we can approximate the *average* energy level at which an electron is present is with the Fermi-Dirac distribution:

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

where E is the energy level, k is the Boltzmann constant, T is the (absolute) temperature, and E_F is the **Fermi level**. The **Fermi level** is defined as the chemical potential of electrons, as well as the (hypothetical) energy level where the probability of an electron being present is 50%.

The no. of one particle states lying between momentum p and p + dp is given as –

$$g_p dp = g_s \frac{4\pi p^2 dp}{h^3} \quad \text{-------------- (2)}$$

where g_s = (2S + 1) is called spin degeneracy factor, arising due to spin of fermions. Since $p^2 = 2mE \Rightarrow 2p\,dp = 2m\,dE \Rightarrow dp = \frac{m}{p} dE = \sqrt{\frac{m}{2E}} dE$.

So, no. of states lying between energy range E and E + dE is given as –

$$g(E)\,dE = g_s \frac{4\pi v}{h^3} 2mE \sqrt{\frac{m}{2E}} dE \quad \text{-------------- (3)}$$

where g (E) is called density of states function.

The total no. of energy states lying between 0 and specified value E_F (Fermi energy) is given as -

$$g_f = g_s \frac{4\pi v}{h^3} \left(2m^3\right)^{1/2} \int_0^{E_E} E^{1/2} dE = g_s \frac{4\pi v}{h^3} \left(2m^3\right)^{1/2} \frac{E_F^{3/2}}{3/2} = g_s \frac{4\pi v}{3h^3} \left(2mE_F\right)^{3/2} \quad \text{-------------- (4)}$$

Further, in F – D distribution, not more than one particle can occupy a given cell which is also obvious from –

$$n_i = \frac{g_i}{D e^{E_i/kT} + 1}$$

at T = 0, we have D = 0 So, $n_i = g_i$

Therefore, taking $g_f = n$, we can write –

$$n = g_s \frac{4\pi v}{3h^3}(2mE_F)^{3/2} \quad \Rightarrow \quad E_F = \frac{h^2}{2m}\left(\frac{3n}{4\pi v g_s}\right)^{2/3}$$ ------------
-- (5)

For an electron gas – S = 1/2 so, g_s = 2S + 1 =2

Therefore, $$\boxed{E_F = \frac{h^2}{2m}\left(\frac{3n}{8\pi v}\right)^{2/3}}$$ ------------
-- (6)

This is the expression for the Fermi energy of an electron gas.

You can calculate the fermi energy state using:

$$E_f = (\frac{h^2}{8m})(\frac{3N}{\pi V})^{\frac{2}{3}}$$

N - number of possible quantum states

V - volume

m - mass of electron

h - planc's constant

Calculating Fermi Energy

To determine the lowest possible Fermi energy of a system, we first group the states with equal energy into sets and arrange them in increasing order of energy. We then add particles one at a time, successively filling up the unoccupied quantum states with the lowest energy.

When all the particles are arranged accordingly, the energy of the highest occupied state is the Fermi energy. In Spite of the extraction of all possible energy from metal by cooling it to near absolute zero temperature (0 Kelvin), the electrons in the metal still move around. The fastest ones move at a velocity corresponding to a kinetic energy equal to the Fermi energy.

The Fermi level and Fermi energy are usually confusing terms and often used interchangeably to refer each other. Although, both the terms are equal at absolute zero temperature, they are different at other temperature.

Fermi energy is applied in determining the electrical and thermal characteristics of the solids. It is one of the important concepts in superconductor physics and quantum mechanics. It is used in semiconductors and insulators.

Fermi Energy Level

The reason for the existence of this energy level is due to Pauli's exclusion principle which states two fermions cannot occupy that same quantum state. So, if a system has more than one fermion, each fermion has a different set of magnetic quantum numbers associated with it.

The Fermi Temperature can be defined as the energy of the Fermi level divided by the Boltzmann's constant. It is also the temperature at which the energy of the electron is equal to the Fermi energy. It is the measure of the electrons in the lower states of energy in metal.